Suppression of the edge interchange instability in a straight tokamak

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A shear-flow-ballooning transformation is used to demonstrate that the presence of a poloidal shear flow can suppress the ballooning interchange instabilities in a high-β straight tokamak. It is shown that, with the toroidicity ignored, complete suppression of these instabilities requires the coexistence of both finite poloidal Alfvén velocity shear and flow shear, in such a way that the latter is greater than the former. The DIII-D parameters [Gohil *et al.*, Phys. Rev. Lett. **61**, 1603 (1988)] are used to compare the experimental results and the above prediction, and an agreement is found. [S1063-651X(96)07811-7]

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Deterioration of magnetic confinement in present day tokamaks is often caused by turbulence at the edges [1]. However, during the high-confinement (H mode) discharge the confinement time may increase by a factor of several, indicative of reduced turbulent transport at the edges. In striving for understanding the underlying physics of the H mode, investigations have focused on the connection of increasing shear flow strength and the decreasing turbulence levels [1–3]. Despite the fact that the origin of edge turbulence remains an open question, it is generally believed that edge turbulence tends to have (1) short wavelengths across the field lines and long along the field lines, (2) large fluctuations in the plasma density and, in high-β plasmas, also in the magnetic fields.

Longer (than the ion Larmor radius) wavelength edge fluctuations may arise from the ballooning interchange magnetohydrodynamic (MHD) instability in the high-β regime of operation [4–6]. Indeed, both sizable magnetic and density fluctuations have been observed in the edges of DIII-D [6]; moreover, evidence of strong correlation between suppression of the edge magnetic and density fluctuations and the H mode has also been established [5].

On the other hand, shorter wavelength fluctuations may be due to drift waves and their variations. These perturbations are primarily electrostatic and of low frequency, and are observed in the low-β regime of operation. One of the promising scenarios involving the drift waves is based on the convective secular instabilities as the waves propagate toward the tokamak edge from the core. It has been shown that when a strong shear flow near the edge is present, the flow can reflect the drift waves back to the core, thereby sheltering the edge from the large disturbances of the convectively unstable waves [7].

In this paper, we will address the stabilization of the remaining high frequency edge magnetic fluctuations originated from the MHD ballooning modes [6]. The motivation for this paper arises from the realizations that the poloidal flow shear is comparable to the poloidal Alfvén velocity shear near the edges of the tokamaks during the low-confinement→high-confinement ($L\rightarrow H$) transition, the numerical estimates of which are given at the end of this paper, and that the edge-localized modes (ELMs), responsible for the temporary deterioration of the H mode into the L mode, are believed to be driven by the pressure gradients [6]. We show that the presence of both a strong shear flow and sheared magnetic field is essential in suppressing the interchange instability. Such suppression can be interpreted as a result of phase mixing both by differential convection and by oppositely propagating Alfvén waves.

Some preliminary notions of the parameter regime are now placed in order. The thermal pressure near the edges of tokamaks is much smaller than the axial field pressure ($\beta_z \equiv 8\pi P/B_z^2 \ll 1$), and the ratio of the azimuthal field also much smaller than the axial field ($\epsilon \equiv B_p/B_z \ll 1$). Since one is interested in some radially localized region, a strip near the magnetic resonant surface where the ballooning mode locates, the azimuthal magnetic field B_p and the shear flow $V_y\hat{\mathbf{y}}$ may be expanded in the Taylor's series up to the linear terms in radius: $B_p(x)=B_y+B_y'x$ and $V_y(x)=Ax$. We may thus use the reduced MHD scaling to approximate the perturbations of the field and the flow, with the latter having a small component along the axial direction [8]. The primary instability driving source is the pressure gradient for the ballooning modes and thus included in the appropriate reduced MHD equations is the leading order pressure gradient term. To this order, the plasma motion can be assumed incompressible. The toroidal field curvature is ignored for simplicity, but we will return to discuss its importance in the end of this report.

We assume that the perturbations are proportional to $\exp(-i\omega t+ik_y y+ik_z z)$ with $k_z/k_y \sim -B_z/B_y$. Combining the preceding considerations, it is straightforward to derive the linearized equations which can be cast into a second-order ordinary differential equation for the stream function ϕ of the perturbed velocity perpendicular to the magnetic field ($\delta\mathbf{v}\equiv\hat{\mathbf{b}}\times\nabla\phi$ and $\hat{\mathbf{b}}$ the local field direction) [9,10]

$$\frac{d}{dx}\left((\omega-k_yAx)^2-\frac{(\mathbf{k}\cdot\mathbf{B})^2}{4\pi\rho}\right)\frac{d\phi}{dx}-k_y^2\left((\omega-k_yAx)^2-\frac{(\mathbf{k}\cdot\mathbf{B})^2}{4\pi\rho}\right)\phi-\frac{2k_z^2(-P_0')}{r\rho}\phi=0, \quad (1)$$

where $P_0'\equiv dP_0/dr$. We have ignored the higher-order terms in the expansion of small β_z, so that slow and fast magnetosonic waves have been eliminated.

The difficulty associated with a differential equation such as Eq. (1) is that there exists singular points when

$\omega = k_y Ax \pm \mathbf{kB}/4\pi\rho$. Physically, these correspond to the wave-flow resonances, and often, but not always, lead to wave damping. It is unclear how, and to what extent, the wave-flow resonances can affect a system that has an intrinsic instability driving source such as the bad curvature in this case. In Sec. I, a shear-flow-ballooning representation is introduced that helps resolve this difficulty.

I. SHEAR-FLOW-BALLOONING TRANSFORMATION

Ballooning representation [11] of the MHD perturbation was originally invented to circumvent a similar difficulty mentioned in the last paragraph. It is an eikonal approximation near the magnetic resonance surface ($\mathbf{k}\cdot\mathbf{B}=0$), where the fast radial variation is separated from the slow dynamics along the field line. In addition, within a thin strip near the magnetic resonance surface where this approximation holds, one finds it more convenient to adopt a Cartesian coordinate, where x is perpendicular to the flux surface, y in the poloidal direction, and z in the toroidal direction.

In the presence of a shear flow, we may adopt, in addition to the ballooning representation, a shear-flow coordinate [12–15]. The fast variations of the perturbation in space and time can be extracted with the phase factor $\exp[ik_x x + ik_z(z + xyB_z B_y'/B_y^2) - ik_y Axt]$. The slow variation is contained in another amplitude factor $g(y,t)$, which depends only on the coordinate y and the time t in this eikonal representation. That is, $\phi(\mathbf{x},t) = g(y,t) e^{ik_x x + ik_z(z + xyB_z B_y'/B_y^2) - ik_y Axt}$. In this representation, convertion of the differentiation and the variables goes as follows: $-iw \rightarrow -ik_y Ax + \partial/\partial t$, $ik_y \rightarrow -ik_z B_z/B_y(x) + \partial/\partial y$ and $d/dx \rightarrow i(k_x - k_y At + k_y B_y' y/B_y)$. Note that when the flow shear A is absent, we recover the usual ballooning transformation.

Thus Eq. (1) is transformed to become

$$\frac{\partial}{\partial t}\,[(a_x + At + Sy)^2 + 1]\,\frac{\partial g}{\partial t} - V_{Ay}^2\,\frac{\partial}{\partial y}\,[(a_x + At + Sy)^2 + 1]\,\frac{\partial g}{\partial y} - \left(\frac{2a_z^2(-P_0')}{r\rho}\right) g = 0, \tag{2}$$

where $S \equiv -B_y'/B_y$, $V_{Ay}^2 \equiv B_y^2/4\pi\rho$, $a_x \equiv -k_x/k_y$, and $a_z \equiv k_z/k_y$. Apparently, this equation is a hyperbolic partial differential equation describing the wave propagation along the field lines; most noticeably, Eq. (2) contains no apparent wave-flow resonances. Reconstruction of the periodicity of the perturbation around the minor radius (y) demands that g vanishes at $y = \pm\infty$ at every instant of time [11]. As will be shown later, this requirement is crucial in differentiating the unstable modes from the stable ones.

The variable coefficients in Eq. (2) depend only on a particular combination of y and t. It is more convenient to define new coordinates (τ,η) in place of (t,y), where $\tau \equiv t - Sy/A$ and $\eta \equiv t + Sy/A + a_x/A$. Thus, we may extract $e^{\gamma\tau}$ from the perturbation: $g(\tau,\eta) = h(\eta)e^{\gamma\tau}$, where $\gamma > 0$. The remaining amplitude factor $h(\eta)$ satisfies the following equality:

$$\left(\frac{d^2}{d\eta^2} + \left[\gamma^2(1-J^2) - \frac{\eta_0^2}{\eta^2+\eta_0^2} + \frac{2a_z^2(-P_0')}{r\rho(\eta^2+\eta_0^2)(1-R^2)S^2V_{Ay}^2}\right]\right) f(\eta) = 0, \tag{3}$$

where $f(\eta) \equiv h(\eta)\sqrt{\eta^2+\eta_0^2}\exp(-\gamma J\eta)$, $R \equiv |A/SV_{Ay}|$, $J \equiv (1+R^2)/(1-R^2)$ and $\eta_0^2 \equiv 1/A^2$. By setting $\gamma = 0$, one obtains an equation similar to the familiar one that gives rise to the Suydam's criterion for the interchange instability. At first glance, a generalized Suydam's criterion may seemingly apply, where the existence of an exponentially growing localized mode requires that $(-a_z^2 P_0')/r\rho(1-R^2)S^2V_{Ay}^2 > 1/8$ in the limit $\eta_0 \rightarrow 0$. However, the above condition only warrants localization of the modes in the new coordinate η, and not necessarily in the actual spatial coordinate y. To satisfy mode localization in y, further consideration is needed. Provided that the function $f(\eta)$ is spatially bound, the asymptotic behavior of $h(\eta)$ for large η is $e^{\gamma\eta(J \pm \sqrt{J^2-1})}$. Together with the factor $e^{\gamma\tau}$, the behavior of $g(\tau,\eta)$ for large y goes as $e^{\gamma(\kappa+2)t + \gamma\kappa y/R}$, where

$$\kappa \equiv 2\left(\frac{R^2}{1-R^2} \pm \left|\frac{R}{1-R^2}\right|\right).$$

A straightforward algebra shows that localization of $g(\tau,\eta)$ in y requires that the parameter

$$R^2 < 1, \tag{4}$$

i.e, $A^2 < (B_y')^2/4\pi\rho$, or $A^2 < (V_{Ay}')^2$. That is, the flow shear must be smaller than the Alfvén velocity shear. Note that when $R^2 < 1$, we have $\kappa + 2 > 0$ and the exponent of the time dependence in g is always positive. It is only for a sub-Alfvénic velocity shear can the perturbations be bounded in space and grow exponentially in time. This conclusion is independent of the sign of the flow shear relative to the Alfvén velocity shear. When the flow shear becomes super-Alfvénic, the eigenmode analysis fails, spatially bound discrete modes are absent and the perturbations in such a system form a continuum, yielding algebraic time dependence.

II. TRANS-ALFVÉNIC FLOW SHEAR ($R^2 \leqslant 1$)

In fact, there exists an additional condition to ensure spatially bound solutions. In deriving inequality (4), we have assumed that the function $f(\eta)$ is spatially bound, which is not automatically satisfied for any value of γ. Existence of a bound $f(\eta)$ requires that the term in the square bracket in Eq. (3) be positive at $\eta = 0$, which in turn imposes a condition for the value of γ not to exceed an upper bound $\gamma_{\max} = Q\sqrt{1-R^2}/R$, where $Q \equiv \sqrt{(-a_z^2 R^2 P_0')/2r\rho}$. The above expression is evaluated using the limit $\eta_0 \rightarrow 0$. The standard method of the WKB quantization for bound solutions can be used for an accurate evaluation of γ.

We shall now investigate the regime when $R^2 \rightarrow 1$ from below in an attempt to understanding the nature of the Alfvénic transition. Unstable perturbations consist basically of two Alfvén waves propagating along the field lines oppositely. Due to the flow convection, the asymptotic spatial structure of the perturbation appears very different on either side of the expanding wave packet. On the wave front parallel to the flow ($y>0$), the propagation leading edge is sharp, behaving as $e^{-Qy/\sqrt{1-R^2}}$. By contrast, on the trailing side ($y<0$) the wave form is smoothly varying as $e^{2Qy\sqrt{1-R^2}}$. The width of the leading edge scales as $(1-R^2)^{1/2}$ and when $R^2=1$, the leading edge becomes infinitely sharp, a shocklike singular structure. Hence when R^2 passes beyond unity such a solution can no longer exist. The discrete normal modes are lost and the only solutions are in the continuum to be discussed later.

Using the terminology of relativity, one may understand the loss of discrete spectra to be due to a loss of casual contact by Alfvén waves within the perturbed region. For a sub-Alfvénic flow shear, the coordinate η remains a spacelike variable, and perturbations within the domain can still communicate. However, for a super-Alfvénic flow shear, the coordinate η becomes timelike and τ spacelike; perturbations are convected away by the rapid flow and communications between perturbations on either side of the wave packet become impossible. This point is manifested by the absence of a resonant cavity in selecting discrete normal modes. Without a resonant cavity, the originally unstable modes in the continuum must be phase mixed, thereby greatly reducing its ability to grow.

III. SUPER-ALFVÉNIC FLOW SHEAR ($R^2>1$)

With τ becoming spacelike and η timelike for a super-Alfvénic flow shear, one may extract the sinusoidal dependence $e^{i\omega\tau}$ from $g(\tau,\eta)$ by replacing γ by $i\omega$, corresponding to wavy perturbations along the field lines. In fact, for a finite ω, one may easily show, by solving Eq. (3), that the asymptotic behavior of $h(\eta)$, except for the sinusoidal dependence, goes as η^{-1} at large η. For any finite time, h is localized in y and therefore such a solution satisfies the ballooning boundary condition in recovering the periodic solution around the minor radius. These solutions represent neutrally stable traveling wave packets.

A special case for the super-Alfvénic flow shear is the limit where $\gamma=0$. Again, Eq. (3) has no singularity except for $\eta\to\infty$. At large η, Eq. (3) reads

$$\left(\frac{d^2}{d\eta^2}-\frac{W}{\eta^2}\right)f=0, \tag{5}$$

where $W\equiv 4Q^2/(R^2-1)>0$ for $R^2>1$. It follows that $h(\eta)=\eta^{-1}f(\eta)\sim\eta^{\alpha}$, where $\alpha\equiv-0.5(1\pm\sqrt{1+4W})$. These solutions are either monotonically increasing in space and therefore unphysical, or bound in both space and time. The latter is again a stable traveling solution.

Finally it is instructive to consider both limits: $\omega=0$ and $S\to 0$. This situation exists when the magnetic resonant layer ($S=0$) has a finite width. One finds that there is no mixing between the space coordinate y and time t in Eq. (2). The solution has algebraically growing time dependence [16,17]. The indicial index becomes $\alpha=0.5(\sqrt{1+4W_0}-1)$, where $W_0=2a_z^2(-dP_0/dr)/r\rho A^2$ and the kinetic energy density grows as $t^{2(\alpha+1)}$, contributed primarily by the y component of the perturbed velocity.

However, how can this solution reconcile with the ballooning boundary condition that g vanish at $y\to\pm\infty$? Note that when $S=0$, the only y dependence in ϕ is through $e^{ik_y y}$, which is in itself periodic (in y) around the minor radius. Hence, there is actually no need to employ the ballooning transformation in this case and the condition that g vanish at $y=\pm\infty$ required by the ballooning formulation is thus avoided.

The limit $S=0$ demonstrates an important point. It shows that without magnetic shear the shear flow alone cannot completely stabilize the interchange instability. Assistance from the magnetic shear for stabilizing the algebraic growth may not be intuitively obvious since even a tiny amount of magnetic shear can help stabilization. The key physics underlying the magnetic-shear-assisted stabilization has to do with the additional phase mixing caused by oppositely propagating Alfvén waves along the field lines. Even with a tiny magnetic shear, considerable wave mixing at the resonant surface can be produced, thereby suppressing the weak algebraic growth. The region $S=0$ of a finite width is located where the toroidal current vanishes and is about to reverse the sign. In this circumstance, one recovers the well known example of Rayleigh-Taylor instability in an unstably stratified atmosphere under the wind shear [16].

In sum, we have shown that linear shear flows can stabilize the MHD interchange instability, consistent with the previous works where the toroidal shear flows are considered [12–15]. To achieve stabilization, we find that two conditions must both be met. First, the flow shear must be sufficiently strong, so strong that it is greater than the poloidal Alfvén velocity shear. Second, the poloidal Alfvén velocity shear, however, cannot be arbitrarily small. When the first condition is not met, the interchange modes remain exponentially unstable although their growth rates are reduced by the shear flow. When the second condition is not satisfied, the interchange modes can, though significantly suppressed, still become algebraically growing.

To gain a better idea as to how suitable the present analysis may apply to the existing tokamaks, we take the parameters of the DIII-D tokamak [6] as a typical example. During the $L\to H$ transitions of the DIII-D, the electron number density is about 4×10^{13} cm^{-3}, the toroidal field 2×10^4 G, and the toroidal current 1 MA. With the minor and major radii about $a\sim65$ cm and $R\sim170$ cm, respectively, and the effective ion mass ~4 m_p, the poloidal Alfvén velocity shear SV_{Ay} is estimated to be about $(1.5–2)\times10^6$ sec^{-1}. On the other hand, the poloidal flow shear A is measured at about 1.6×10^6 sec^{-1} within a spatial range of 2.5 cm centering around the location 1.5 cm interior of the plasma edge [18]. These estimates show that the edge of DIII-D is approximately in the regime where $R\sim1$, the marginally stable, trans-Alfvénic shear regime. It is of interest to note that during the stable operation of the H mode, the edge electron density increases in time [3] as a result of better confinement. According to the preceding analysis, this tendancy decreases the Alfvén speed and pushes the edge plasmas into an even more stable regime, thereby sustaining a long-lived (>100 ms) H mode.

The present work, however, does not include the effects of toroidicity. It hence may not be compared, in quantitative details, with the stabilization of edge tokamaks during the high-β H-mode discharge. Nevertheless, based on the idea developed in this paper that differential convection introduces stabilizing *phase* mixing, we expect that convection around the poloidal direction also should give rise to additional stabilizing *spatial* mixing in a toroidal configuration. Perturbations in the bad curvature region can no longer stay localized and must be convected into the good curvature region; that is, spatial mixing can average out the negative and positive potential energy, thereby reducing the instability driving source for an otherwise localized perturbation. We therefore anticipate that the toroidicity can somewhat strengthen the stability criterion $R>1$.

The above notions of convective stabilization have actually been demonstrated in a previous work addressing the effects of the toroidal shear flows in toroidal devices by adopting the same shear-flow-ballooning representation. [15]. Upon including the toroidicity in the presence of a toroidal flow, the analysis must take into account the plasma compressibility [14]. As a result of the rich physics involving the toroidicity and compressibility, extensive numerical integration must be employed in the studies formulated as an initial-valued problem [15], in contrast to the present simpler case for which analytical quantitative results can be obtained and the physics of flow shear explicitly demonstrated. It turns out that not only do the numerical results yield consistent criteria (modified to incorporate with the toroidal flow) for the flow-shear stabilization as the one obtained here, but the aforementioned convective stabilization is also clearly shown by observing in the results periodic bursts of perturbations around the torus.

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